Continuous Spectrum, Characteristic Modes, and Leaky Waves of Open Waveguides

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Abstract—**The modes of open waveguides with nonseparable cross sections are derived by means of an extension of the resonance equation for the electromagnetic field. Such modes, forming a continuous spectrum, allow us to apply to discontinuity problems in open environments the techniques originally developed for closed waveguides.**

In this paper, the resonance equation is generalized according to functional analysis considerations. By this approach, it is possible to derive the modal spectrum from the simultaneous diagonalization of the real and imaginary parts of the admittance of the structure. A variational interpretation of the solution of the generalized resonance equation gives additional insight into the modes of open waveguides.

The generalized resonance equation, when applied to three-dimensional objects, provides the well-known characteristic modes of these structures. The relationship between continuous spectrum, characteristic modes, and leaky waves is also discussed.

I. Introduction

The modes of closed waveguides of separable cross section (e.g., rectangular, cylindrical, etc.) are well known from basic microwave courses [1]. These modes are real solutions of the two-dimensional Helmholtz equation considered in the plane transverse to the propagation direction; since the waveguide is closed, the eigenvalues of the above equation form a discrete set. A similar situation also occurs for three-dimensional closed resonators. When the boundary surfaces of the resonator are coordinate surfaces, the three-dimensional Helmholtz equation can be solved in closed form, and again a discrete spectrum of eigenvalues is present.

For closed waveguides, or resonators, of nonseparable geometry, the solution procedure of the Helmholtz equation is still well known, even if it is more involved. In these cases, in fact, modes can be obtained by a transverse resonance procedure. As an example, let us examine the ridged waveguide of Fig. 1. In this case, by using the symmetry, we can consider just one-half of the structure. We choose as unknowns the tangential components of the electric field E on the separation surface Σ_a. By considering suitable admittance operators, the magnetic field is expressed in terms of the tangential components of E. In each of the two regions, that is, the regions below and above Σ_a, we have $H_1 = Y_1(E), H_2 = Y_2(E)$. After equating on Σ_a the tangential components of the magnetic fields, and by discretization, we obtain a matrix which depends on the transverse wavenumber k_t. The zeros of the determinant of this matrix can be obtained for certain values of k_t and they correspond to the resonance of the structure. Equivalently, we can find the eigenvalues of the above matrix; for certain values of k_t, an eigenvalue is zero, then the determinant is zero, and the corresponding eigenvector provides the modal field distribution. Following this transverse resonance procedure, we can rigorously obtain the modal spectrum of closed, nonseparable, structures.

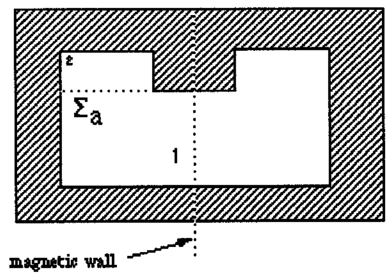

Fig. 1. Example of a closed waveguide of nonseparable cross section.

For open waveguides, apart from a few discrete modes at most, the spectrum becomes continuous (Fig. 2). For open structures, when the coordinate surfaces coincide with the boundaries, the modal spectrum is known. An excellent collection of solutions in the various reference systems is provided by [2]. Let us take a simple one-dimensional example of a dielectric slab waveguide. Its modal spectrum consists of a finite number of discrete (surface wave) modes and a continuous spectrum. The former are standing wave (in the direction normal to the surface) solutions with discrete eigenvalues. The continuous spectrum, on the other hand, has no discrete eigenvalue; but once a particular wavenumber is chosen, the field is a standing wave solution consisting of the incoming wave and the outgoing wave in the direction normal to the dielectric surface. This standing wave remains finite at infinity and does not individually satisfy the radiation condition. A combination of these, however, represents radiation by any physical source, thus satisfying the radiation condition. Hence, such standing waves are "modes" of the structure.

Nevertheless, for open structures of nonseparable cross section it was not well known for a long time how to proceed in order to obtain the spectrum as for the case of separable ones [3], [4]. Most open waveguides currently used, both in microwaves and in optics, fall into the latter category (e.g., microstrips, slot lines, coplanar waveguides, inset waveguides, dielectric waveguides, etc.). In some cases, it has been natural to extend the transverse resonance technique also to open

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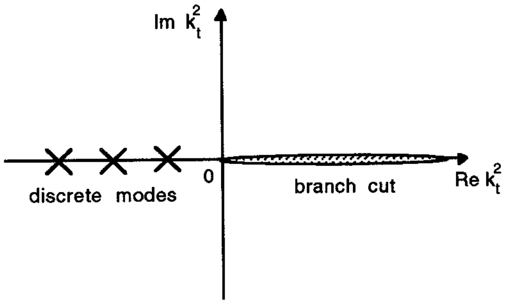

Fig. 2. Complex k_t^2 plane. The branch cut on the real axis of k_t^2 is shown together with a few discrete modes.

waveguides obtaining in this way the "leaky waves." This procedure, which is extraordinarily simple and elegant when applicable, has lead to the understanding of some difficult problems with modest computer resources [5]. In an open environment, however, the Helmholtz equation together with its boundary condition is not self-adjoint. As a consequence, the "leaky waves," which grow at infinity, are not part of the modal spectrum and are not suitable to a global representation of the field on the guide cross section.

Recently, by properly extending the transverse resonance technique, the spectrum of some open waveguides has been obtained [6]–[10]. The importance of such a spectrum is evident; in fact, discontinuities present along the waveguide excite the whole spectrum, therefore generating radiation, surface waves, mode conversion, etc. Moreover, the coupling and the interference of waves incident from the exterior can easily be accounted for if the spectrum is known.

Up to now, we have only considered open waveguides. However, nonseparable three-dimensional objects also possess modal fields. The knowledge of the latter greatly enhances the solution of antenna and scattering problems. The study of the above modes (also called characteristic modes), for three-dimensional objects, has been initiated with the fundamental work of Garbacz [11], [12]. In [13] and [14] the method of moments has been applied in order to obtain a formulation of the problem as well as an efficient numerical algorithm; in [15] the technique has been suitably extended also to apertures in metallic bodies. Successively, characteristic modes have been successfully used for the synthesis and optimization of antennas [16]–[19].

In this paper, we describe the derivation of the continuous spectrum of nonseparable open waveguides. This method, when applied to three-dimensional objects, gives the modal characterization of the structure (characteristic modes) and coincides with the approach developed in [11]–[19], when applied to waveguides (open or closed), it yields the spectrum (both continuous or discrete). Therefore, it represents the sought generalization of the transverse resonance approach to open, nonseparable problems.

II. Theory of the Continuous Spectrum and of the Characteristic Modes

As already mentioned in the introduction, for closed structures it is possible to obtain the discrete set of modes by

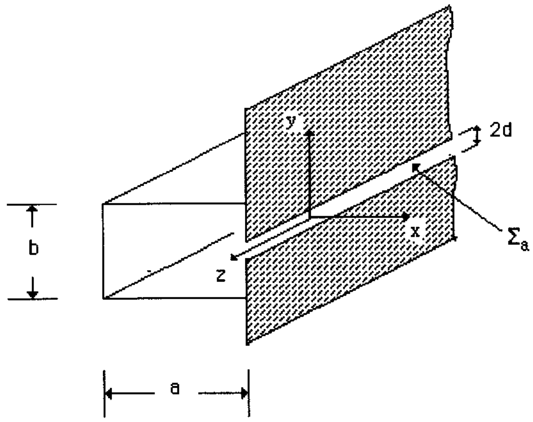

Fig. 3. Slotted waveguide (as an example of an open waveguide of nonseparable cross section). Σ_a is the surface along the slot, while Σ_∞, which is not reported, is the surface at infinity for z = cost.

searching the resonance of the structure. This approach can also be rigorously extended to open structure. To be specific, let us consider the case of a slotted waveguide as in Fig. 3. In this case, the natural formulation of the problem is done in terms of the equivalence theorem and of admittance operators. With reference to Fig. 3, we define the following inner product on Σ_a as:

$$\langle B, C\rangle = \int_{\Sigma_a} B^* C \, dl \tag{1}$$

where the * denotes complex conjugate; moreover, we define an analogous inner product $\langle B, C\rangle_\infty$ on the surface at infinity. Since we are considering a uniform guiding structure, the z dependence has been separated out, and the integrals along Σ_a, Σ_∞ are simple line integrals.

Since the structure is open, radiation phenomena of the electromagnetic energy are, in general, present. Due to radiation, the admittance operator involved is complex. In the following, in order to avoid unnecessary analytical burden, we refer to the scalar TE or TM cases, even though the hybrid case is immediately describable by using dyadics. If we try to account for radiation by using a complex k_z, we also get complex values of k_t in the 2-D Helmholtz equation

$$\nabla_t^2 \tau + k_t^2 \tau = 0 \tag{2a}$$

where $k_o^2 = \omega^2 \mu \varepsilon$; $k_t^2 = k_x^2 + k_y^2 = k_o^2 - k_z^2$, and τ represents the generic field (or potential). On the other hand, it is possible to observe that the continuous spectrum corresponds, in the complex plane (Fig. 2), to a branch cut from the origin to infinity. This branch cut can only be made along the real axis if we want fields finite at infinity. But a branch cut along the real axis corresponds to a real value of k_z, i.e., absence of attentuation in the propagation direction. This absence of attentuation can only be realized if the outgoing waves (radiation from the structure to infinity) are compensated by incoming waves (energy coming from infinity to the structure). This arrangement corresponds to standing waves for the radiative part ($k_t \leq k_0$) of the field. Such an approach has been successfully employed in [6]–[10] to determine the continuous spectrum of open waveguides. In [6]–[10], the characteristics of the standing waves have been described in terms of the phase shift which a plane wave undergoes when it is reflected from, say, the slotted plane (Fig. 3). Then, by equating the tangential components of the magnetic fields on Σ_a, a generalized eigenvalue equation is obtained. The solution of the generalized eigenvalue equation allows the calculation of the continuous spectrum.

It is possible to arrive at the same conclusions in a very direct way, by considering (2a) from the viewpoint of functional analysis, while considering k_t real ($0 < k_t < \infty$). Referring to (2a), the boundary condition corresponding to the Sommerfeld radiation condition (waves propagating toward infinity) is given by

$$\frac{\partial v}{\partial r} - jk_t v = 0 \tag{2b}$$

where v denotes the outgoing part of τ. While (2a) is self-adjoint, condition (2b) it is *not* self-adjoint in the Hermitian sense; in fact, the adjoint boundary condition is

$$\frac{\partial u}{\partial r} + jk_t u = 0 \tag{2c}$$

where u denotes the part of τ which corresponds to a wave coming from infinity toward the guide. Therefore, if we want to use of the eigenfunction expansion method for our diffraction problem, we have to complement our problem with the adjoint problem, which requires waves coming *from* infinity instead of waves going *to* infinity [20, pp. 299–300].

Let us call the waves coming from infinity a, and the waves reflected by our scatterer b. In terms of a scattering operator S we have

$$b = Sa$$

where, obviously, the operator S is unitary but not self-adjoint. Therefore, we have to introduce the following shifted eigenvalue problem [20]:

$$Su_n = \sigma_n v_n$$
$$S^+ v_n = \sigma_n u_n$$

where S^+ is the adjoint of S and u_n, v_n are the eigenfunctions, while σ_n are the singular values. Note that u_n are a basis appropriate to represent the incoming waves (the a); and v_n are a basis appropriate to represent the outgoing waves (the b). Analogous considerations hold for a three-dimensional scatterer if we expand the fields in terms of multipoles correspondent to spherical waves incident (e^{jkr}) and reflected (e^{-jkr}).

As already stated, when apertures on a metallic screen are present, the natural representation of diffraction problems is in terms of admittance operators. For example, with reference to Fig. 3, we can enforce the continuity of the tangential components of the electric field by considering appropriate magnetic currents. Accordingly, our problem has been divided into two parts—a region corresponding to the waveguide, and a region corresponding to the air half-space. We can express the magnetic field inside each region in terms of the magnetic currents, and then we can equate the tangential components of the magnetic field on the aperture Σ_a. The equation obtained in this way contains an admittance operator Y and the sum of the two operators corresponding to the two regions, which is not self-adjoint. Let us separate its real and imaginary parts

$$Y = G + jB \tag{3}$$

where G, B are symmetrical, real, and self-adjoint; they can be obtained by Y and by its complex conjugate Y^*, as

$$G = (Y + Y^*)/2 \qquad B = (Y - Y^*)/2j. \tag{4}$$

When sources are present, the general radiation/scattering problem can be described in terms of the operator Y as

$$Y(\phi) = \Psi$$

where ϕ represents the field on Σ_a and Ψ is a generic excitation. Note that Y depends on the geometry of the problem, dielectric constants, and on k_t. Note also that, since G, B are self-adjoint, they admit a spectral decomposition of the following type:

$$G\xi_k = \mu_k \xi_k \qquad B\zeta_k = \nu_k \zeta_k$$

where μ_k, ν_k are the eigenvalues (reals and positives) and ξ_k, ξ_k are the eigenfunctions. Naturally, the inner product $\langle \xi_k, G\xi_k \rangle$ corresponds to the radiated energy, while $\langle \zeta_k, B\zeta_k \rangle$ corresponds to the reactive energy. The two operators, being real symmetric and G being positive definite, can be diagonalized simultaneously by a common basis of eigenfunctions. By denoting with ϕ_n both ξ_n and ζ_n, we can write

$$G\phi_n = \mu_n \phi_n \qquad B\phi_n = \nu_n \phi_n$$

by substituting, we obtain

$$\chi_n G\phi_n = B\phi_n \tag{5}$$

where $\chi_n = \nu_n/\mu_n$. Equation (5) is the generalized transverse resonance condition for open structures (nonself-adjoint problems). From (5), for each k_t value, it is possible to obtain a discrete number of eigenfunctions $\phi_n (n = 1, 2, \cdots)$ which represent the modes of our structure. Observe that in (5) only the ratio between ν_n and μ_n is used. The possibility of normalizing differently the quantity $\langle \phi_n, G\phi_n \rangle$ corresponds, for the three-dimensional case, to the different choices adopted in [16]–[19]. In the following, the choice $\langle \phi_n, G\phi_n \rangle = 1$ has been adopted. Since G, B are real, symmetrical, operators all the eigenvalues χ_n are real, and all the eigenfunctions ϕ_n can be chosen to be real. The eigenfunctions, when suitably normalized, satisfy the following orthogonality relationship:

$$\langle \phi_m, G(\phi_n) \rangle = \delta_{mn} \tag{6a}$$
$$\langle \phi_m, B(\phi_n) \rangle = \chi_n \delta_{mn} \tag{6b}$$
$$\langle \phi_m, Y(\phi_n) \rangle = \lambda_n \delta_{mn} \tag{6c}$$

where δ_{mn} is Kronecker delta ($\delta_{mn} = 0, m \neq n; \delta_{mn} = 1, m = n$) and $\lambda_n = 1 + j\chi_n$. By using the generalized Green's theorem [21, pp. 870–874], it is also possible to show that

$$\langle \phi_m, G(\phi_n) \rangle_\infty = \delta_{mn}. \tag{6d}$$

Therefore, the eigenfunctions simultaneously diagonalize the operators G, B, Y. It is also noted that the solution of (5) minimizes the following Rayleigh ratio:

$$\chi = \frac{\langle \phi, B(\phi) \rangle}{\langle \phi, G(\phi) \rangle}. \tag{7}$$

This functional corresponds to the ratio between the stored reactive energy and the radiated energy. Accordingly, the eigenfunctions are the field distribution which minimizes the ratio between the reactive and radiative energy. In terms of the quality factor Q, this corresponds to saying that Q is minimum; or, in terms of Lagrangians, minimization of (7) gives a minimum of the Lagrangian of the system. Therefore, for open waveguides, to each k_t value corresponds a multiplicity of eigenvalues and eigenfunctions; each eigenfunction describes a different field distribution on the discontinuous interface and generates a field which is the solution of (2a); these fields are the modal fields. The modal solutions, denoted as $\tau_n(r; k_t)$ in the following, are easily calculated from the eigenfunctions. At a given frequency, part of the spectrum is radiating ($k_t < k$), while the remaining part contributes only to the reactive energy ($k_t > k$). Moreover, it is possible to show that modes correspondent to different k_t are orthogonal; that is, in the section S, comprising the guide and the half-space, we have

$$\int_S \tau_n(r; k_t)\tau_m(r; k_t')\, dS = \delta_{nm}\delta(k_t - k_t'). \tag{8}$$

However, the modes are also complete, since

$$\int_0^\infty \sum_n \tau_n(r; k_t)\tau_n(r'; k_t)\, dk_t = \delta(r - r'). \tag{9}$$

The completeness follows from the fact that the operators G, B are self-adjoint and G is positive definite [21, pp. 774–778]. Actually, G is only positive semidefinite, but when it becomes zero, no radiation is present and the classical formulation used for closed structures can be used.

Relationships (6), (8), and (9) allow modal expansion of a given field on a section $z = \text{cost}$ in a manner similar to that employed in closed waveguides. They represent the starting point to treat discontinuity problems in open structure by modal analysis.

A. Characteristic Modes for Three-Dimensional Objects

The above modal theory is, however, also applicable to three-dimensional objects. Actually, it has been introduced in [11] for studying scattering phenomena. The clear exposition of the theory in the framework of functional analysis, as well as its method of moment discretization, has been developed in [13]–[15]. What has not been noticed in these papers is the possibility of obtaining the continuous spectrum of open waveguides. Equivalently, if the theory of characteristic modes is applied to a two-dimensional problem for all values of real $k_t (0 \leq k_t < \infty)$, the complete modal spectrum is obtained. To conclude, the spectrum of two-dimensional or three-dimensional structure can be obtained from a resonance equation. In closed environments, the zeros of the eigenvalues correspond to the natural frequencies of the structure. In open environments, (5) becomes the resonance equation, and for each value of k_t (for 2-D problems) or k (for 3-D problems) we have a set of eigenvalues (characteristic values) and eigenvectors (characteristic vectors) which provide the modal characterization of the structure.

B. "Leaky Waves"

In the case of "leaky waves", the procedure consists of determining a k_t value such as

$$Y\phi_n = \lambda_n \phi_n; \qquad \lambda_n = 1 + j\chi_n = 0. \tag{10}$$

However, this cannot happen for real values of k_t since, for real k_t, χ_n is also real. Therefore, one is forced to consider complex values of k_t corresponding to waves growing toward infinity (nonmodal solutions). It should also be observed that, while the transverse resonance for closed waveguides is a rigorous procedure, when it is applied as in the leaky wave formalism it becomes only an approximation. As stated in [5], this approximation holds if the following are true.

1) *The geometry of the antenna is capable of supporting a wave of complex type.* This is equivalent to saying that at least a solution of (10) exists.

The field contribution due to a complex eigenvalue is predominant in the near field. This condition corresponds to saying that the near field is representable principally in terms of the leaky wave. It should be noted that, rigorously speaking, the field has to be expanded in terms of the continuous spectrum as

$$\int_0^\infty \sum_n A_n(k_t)\tau_n(r; k_t)\, dk_t$$

where $A_n(K_t)$ is the modal amplitude. However, this integral can be deformed in the complex k_t plane. When the contribution to the above integral is principally due to the pole corresponding to the leaky waves, then condition 2 holds and the leaky wave formalism become a useful approximation.

III. Computation of the Continuous Spectrum

Having recognized the relationship between a continuous spectrum and the characteristic modes, all the results developed for the latter can be almost immediately employed to determine the spectrum of open waveguides and vice versa. As an example, in [22] and [23], the case of a slotted screen (Fig. 4) has been studied in terms of the characteristic modes. On the other hand, in [6], the case of the slotted guide of Fig. 3 has been studied according to the modal developed in [6]–[10]. With minor modification, it is possible to apply the theory developed in [6] in order to study the problem of Fig. 4. We will now compare the results.

The knowledge of the continuous spectrum is correspondent to the knowledge, for each k_t, of the eigenvectors of (5). In this case (Fig. 4), these eigenvalues depend on the geometry but not on the frequency. A rigorous Galerkin procedure has been applied to solve (5) for both the TE and TM case. An example of the convergence of the eigenvalues with respect to the number of basis functions is given in Table I for the

TABLE I
CONVERGENCE OF THE EIGENVALUES OF (5) FOR THE TE CASE. THE ELECTRIC FIELD ON THE APERTURE (CF. FIG. 4) HAS BEEN EXPANDED IN TERMS OF CHEBYSHEV POLYNOMIALS AND N REFERS TO THE NUMBER OF BASIS FUNCTIONS USED

N	χ_1	χ_2	χ_3	χ_4	χ_5	χ_6
1	0.4453					
2	0.4413	0.4453				
3	0.0252	0.4113	1.481			
4	0.0252	0.3033	1.481	9.58		
5	0.0210	0.3033	1.457	9.58	162.57	
6	0.0211	0.3023	1.457	9.51	162.57	5137.82
7	0.0211	0.3023	1.457	9.51	161.74	5137.82
8	0.0211	0.3023	1.457	9.51	161.74	5120.99
9	0.0209	0.3023	1.457	9.51	161.62	5120.99
10	0.0209	0.3022	1.457	9.51	161.62	5115.81

TABLE II
CONVERGENCE OF THE EIGENVALUES OF (5) FOR THE TM CASE THE DRIVATIVE OF THE ELECTRIC FIELD ON THE APERTURE, $\partial E_z/\partial y$, HAS BEEN EXPANDED IN TERMS OF CHEBYSHEV POLYNOMIALS. N REFERS TO THE NUMBER OF BASIS FUNCTIONS USED.

N	χ_1	χ_2	χ_3	χ_4	χ_5	χ_6
1	–.0597					
2	–.0597	–.6092				
3	–.0229	–.6092	–8.1384			
4	–.0229	–.5096	–8.1384	–159.8632		
5	–.0220	–.5096	–7.8127	–159.8632	–5063.6068	
6	–.0220	–.5070	–7.8127	–157.4882	–5063.6068	–243298.4952
7	–.0219	–.5070	–7.7943	–157.4882	–5030.5213	–243298.4952
8	–.0219	–.5065	–7.7943	–157.1237	–5030.5213	–242626.5292
9	–.0219	–.5065	–7.7839	–157.1237	–5016.7864	–242626.5292

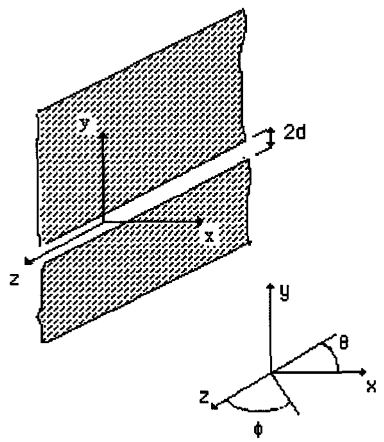

Fig. 4. Geometry of a conducting plane with an infinity long slot of width 2d.

TE case, and in Table II for the TM case. Both cases have been computed for $k_t \cdot 2d = \pi$ (see Fig. 4) and can be compared to the results of [22] and [23]. The comparison shows that, by employing Chebyshev polynomials, very few terms are necessary to achieve a good convergence. Moreover, by increasing the number of terms, the results practically do not change. In Table I, a slight disagreement with the first column of [22, Table I] is noted; this is probably due to the fact that in [22] convergence has not yet been achieved.

In [22] and [23], very useful approximations for the χ_n have been found. These approximations have been checked against the rigorous results and are reported in Fig. 5 for the TE case, and in Fig. 6 for the TM case. In [6]–[10], (5) has generally been solved in terms of the phase shift $\alpha_n(k_t)$ that a plane wave undergoes when it is reflected from the slotted screen. The relationship between $\alpha_n(k_t)$ and χ_n is simply given by $-\cot\alpha_n(k_t) = \chi_n$. It is interesting to note that the oscillations in Fig. 5 are similar to a Fresnel phenomenon (field perpendicular to the edge). It should also be noted that better results are obtained for the approximation relative to the TE case.

Fig. 5 and 6 provide all the information necessary to evaluate the spectrum of the slotted conducting plane. They also describe the electrical behavior of the slotted conducting plane. The phase shift $\alpha_n(k_t) = 90°$ corresponds to $\sin\alpha_n(k_t) = 1$ and is obtained for wide slots in the TE case. Here, the χ_n's are positive since the energy stored near the slot is essentially capacitive. When increasing $k_t \cdot 2d$, $\sin\alpha_n(k_t)$ tends to unity, and therefore χ_n tends to zero. The phase shift $\alpha_n(k_t) = 270°$ corresponds to $\sin\alpha_n(k_t) = -1$ and is obtained for wide slots in the TM case. Here, χ_n's are negative since the energy stored near the slot is essentially inductive.

It is also noted that the first eigenvalue for the TE case is different from zero when k_t is zero. It is therefore possible to find a mode having a zero value of k_t and, as such, propagating just in the z direction. This is the TEM mode of the structure.

IV. CONCLUSIONS

By means of simple considerations of functional analysis, we directly derive a generalization of the resonance equation. This equation allows computation of the continuous spectrum of open waveguides of nonseparable cross section. In the case

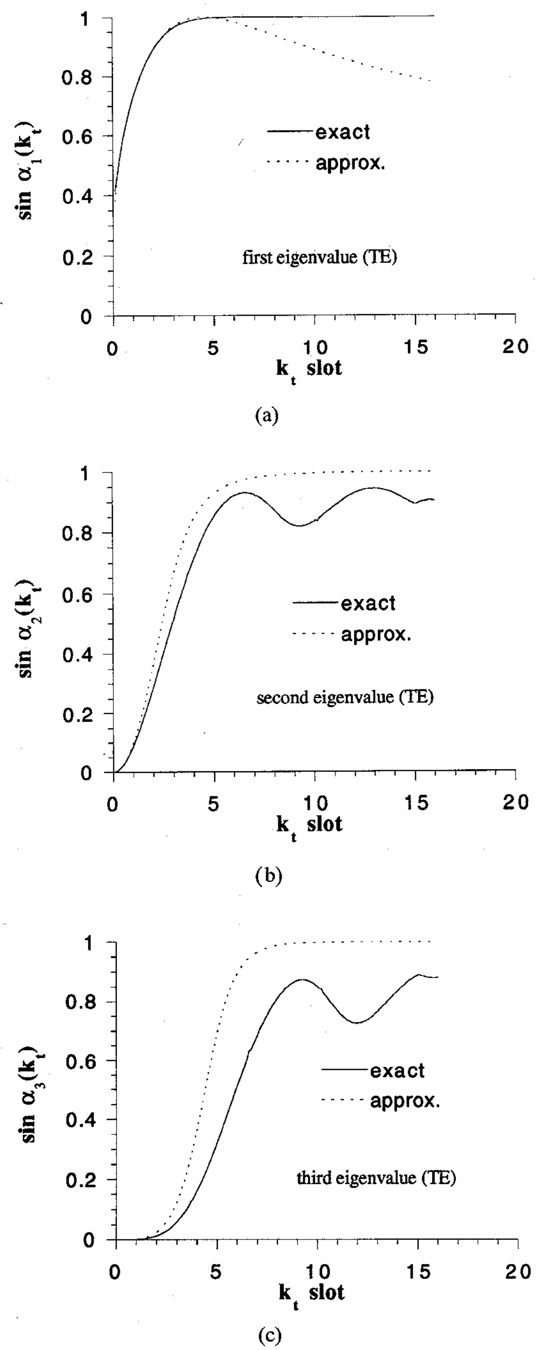

Fig. 5. Behavior of the first three eigenvalues of a slotted screen for the TE case (the slot width is slot = 2d). The continuous curves refer to the rigorous results obtained by considering six Chebyshev basis functions, while the approximate results (dashed lines) are taken from [22].

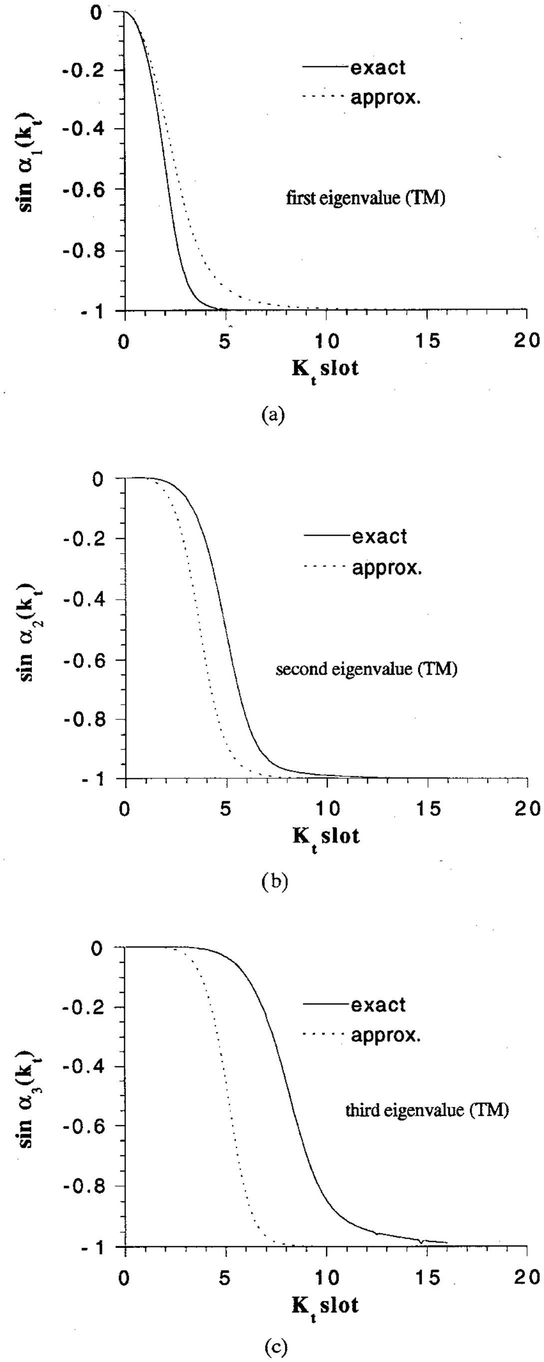

Fig. 6. Behavior of the first three eigenvalues of a slotted screen for the TM case (the slot width is slot = 2d). The continuous curves refer to the rigorous results obtained by considering six Chebyshev basis functions, while the approximate results (dashed lines) are taken from [23].

of three-dimensional objects it corresponds to the already-known characteristic modes.

The generalized resonance equation has been obtained from the simultaneous diagonalization of the reactance and of the radiation conductance by means of a set of common fields. These fields minimize the Lagrangian (admittance) of the system and the ratio between the reactive and radiative energies.

Once the complete modal spectrum is known, modal analysis of open waveguide discontinuities becomes feasible.

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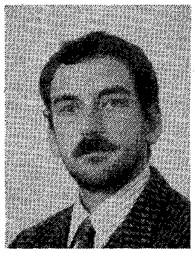

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